What do I need to be able to do?

You should be able to:

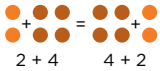
- Understand properties of addition and subtraction.
- Understand properties of multiplication and division.
- Use formal methods of addition and subtraction for integers.
- Use formal methods of multiplication and division for integers.
- Add and subtract directed numbers.
- Multiply and divide directed numbers.
- Understand and use order of operations with positive and negative integers.

Key Words

Commutative	Changing the order of operations does not change the result.
Associative	When you add or multiply you can do so regardless of how the numbers are grouped.
Inverse	The operation that undoes what was done by the previous operation.
Subtract	Taking away one number from another.
Negative	A value less than zero.
Debit	Money that leaves a bank account.
Credit	Money that goes into a bank account.
Integer	A whole number.
Operation	A mathematical process.

Addition

Addition is **commutative**.



2 + 4 = 4 + 2

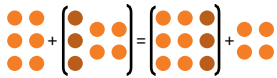
The order of addition doesn't change the result.

Formal Written Method

$$\begin{array}{rccc} & H & T & U \\ & 3 & 4 & 2 \\ + & 1 & 4 & 9 \\ \hline & \mathbf{4} & \mathbf{9} & \mathbf{1} \\ \hline & & 1 & \end{array}$$

Remember the place value for each column!

Addition is **associative**.

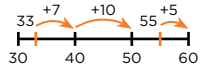


6 + (3 + 4) = (6 + 3) + 4

It doesn't matter how you group the numbers.

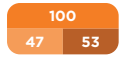
Models to Help with Addition

Number Lines



So we can say
33 + 7 + 10 + 5 = 55
or **33 + 22 = 55**

Bar Models



So we can say
47 + 53 = 100

Part/Whole Diagrams

So we can say
26 + 50 = 76

Subtraction

Subtraction is **not commutative or associative**.

$$12 - 8 \neq 8 - 12$$

When you subtract, **the order must stay the same**.

Formal Written Method

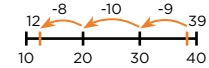
$$\begin{array}{rccc} & H & T & U \\ & 5 & \cancel{3}^{2} & {}^{1}2 \\ - & 2 & 1 & 6 \\ \hline & \mathbf{3} & \mathbf{1} & \mathbf{6} \end{array}$$

Remember 0 is a place holder!

$$\begin{array}{rccc} & 2 & 0 & 8 \\ - & 0 & 0 & 4 \end{array}$$

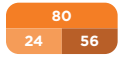
Models to Help with Addition

Number Lines



So we can say
39 - 9 - 10 - 8 = 12
or **9 - 27 = 12**

Bar Models



So we can say
80 - 24 = 56 &
80 - 56 = 24

Part/Whole Diagrams

So we can say
43 - 21 = 22 &
43 - 22 = 21

Written Methods for Multiplication

Long Multiplication

$$\begin{array}{r} 247 \\ \times\ 3 \\ \hline 741 \end{array}$$

Grid Method

X	200	40	7
3	600	120	21

600 + 120 + 21 = **741**

Gelosia

2	4	7	X
0 / 6	1 / 2	2 / 1	3

7 **4** **1**

Repeated Addition

$$\begin{array}{rrrr} & H & T & U \\ & 2 & 4 & 7 \\ + & 2 & 4 & 7 \\ + & 2 & 4 & 7 \\ \hline & \mathbf{7} & \mathbf{4} & \mathbf{1} \\ & 1 & 2 & \end{array}$$

Written Methods for Division

Short Division

$$6\overline{)\,2\,5^{2}\,1^{1}2} \qquad \mathbf{042}$$

$$8\overline{)\,8\,1\,1^{1}6} \qquad \mathbf{102}$$

Short Division (with Remainders)

$$2\overline{)\,2\,5\,1^{1}.\,1^{1}0} \qquad \mathbf{125.5}$$

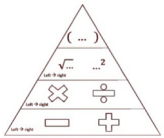
Continue after the decimal point! If you start to get a repeating decimal, stop.

Long Division

$$\begin{array}{rrrrl} & \mathbf{0} & \mathbf{4} & \mathbf{2} & \\ 6\overline{)} & 2 & 5 & 2 & \\ - & 2 & 4 & 0 & \text{6 x 40} \\ \hline & 0 & 1 & 2 & \\ - & 0 & 1 & 2 & \text{6 x 2} \\ \hline & & & 0 & \end{array}$$

This method relies on you being comfortable with multiples of your divisor (in this case, 6!).

Order of Operations



Example 1

(4 x 7) + 3

So we need to evaluate the brackets first; 4 x 7 = 28

This is now 28 + 3 = **31**

Example 2

$(6 + 4 - 3)^2$ x 4

So we need to evaluate the brackets first and we work left to right; 6 + 4 - 3 = 7

This is now
7^2 x 4 = 49 x 4 = **196**

Example 3

4 - 8 x 2 + 12 ÷ 4

So first we do the multiplication/division left to right; 4 - 16 + 3

Now we do the addition/ subtraction from left to right: -12 + 3 = **-9**

Calculations with Directed Numbers

Addition

2 + -3

Remember; If I add a negative, I am adding something that will make it smaller, so it is the same as subtracting that number!

2 -3 = **-1**

Generalisation: **+ - = -** **- - = +**

Subtraction

2 - -3

Remember; If I subtract a negative, I am taking away the amount that was making it smaller, so it is the same as adding that number!

2 +3 = **5**

Multiplication

2 x -3

'2 lots of -3' = **-6**

-2 x -3

Think of this as the negative of 2 x -3 = **6**

Generalisation:

Division

Remember that multiplication and division are inverse operations.

E.g. 6 ÷ -3 = **-2**

-6 ÷ 2 = **-3**

Models to Help

It can be helpful to put calculations involving directed numbers into real life contexts. Think about temperate or bank accounts when unsure.

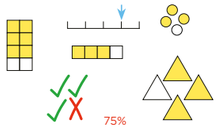
What do I need to be able to do?

You should be able to:

- • Understand different representations of fractions
- Fully simplify fractions
- Recognise and find equivalent fractions
- Convert between mixed numbers and
- improper fractions
- Add/subtract any fractions
- Add/subtract mixed numbers

Representing Fractions

numerator → 3
denominator → 4



We say 'three quarters' or 'three out of four'

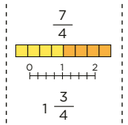
All of these show $\frac{3}{4}$

Mixed Numbers and Improper Fractions

numerator → 3
denominator → 4

$\frac{3}{2}$ $1\frac{1}{2}$

$\frac{10}{3}$ $3\frac{1}{3}$



$\frac{7}{4}$ $1\frac{3}{4}$

Fractions can represent more than one whole.

The denominator tells us how many parts make up one whole

$\frac{9}{5}$

This tells us that one whole is made up of 5 parts. We have 9 parts, so we can make one whole plus 4 parts.

$1\frac{4}{5}$

Key Words

- Numerator: the top number of a fraction
- Denominator: the bottom number of a fraction
- Equivalent: of equal value
- Mixed Number: a number with an integer and a proper fraction
- Improper Fraction: a fraction where the numerator is larger
- than the denominator
- Coprime: two numbers which share no common factors (except 1)

Adding/Subtracting Fractions — Common denominators

$\frac{2}{7} + \frac{4}{7}$

$\frac{2}{7} + \frac{4}{7} = \frac{6}{7}$

Remember that the denominator doesn't change

$\frac{2}{7} + \frac{4}{7}$

$\frac{5}{8} - \frac{4}{8} = \frac{1}{8}$

We can just subtract 4 from 5!

$\frac{1}{10} + \frac{3}{10}$

$\frac{1}{10} + \frac{3}{10} = \frac{4}{10}$

4 and 10 have a common factor (2)

$\frac{2}{5}$ ÷2

You must always fully simplify your fractions

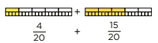
Adding/Subtracting Fractions — Different denominators

$\frac{1}{5} + \frac{3}{4}$

We need to find a common denominator using equivalent fractions

$\frac{1}{5} \times\frac{4}{4} \frac{4}{20}$

$\frac{3}{4} \times\frac{5}{5} \frac{15}{20}$



$\frac{19}{20}$

$\frac{3}{11} + \frac{2}{3} = \frac{9}{33} + \frac{22}{33} = \frac{31}{33}$

the LCM of 3 and 11 is 33, so our equivalent fractions are;

$\frac{3}{11} = \frac{9}{33}$ $\frac{2}{3} = \frac{22}{33}$

$\frac{5}{7} + \frac{4}{9} = \frac{45}{63} + \frac{28}{63} = \frac{73}{63}$

Let's convert it to a mixed number

Remember you can find the LCM of 7 and 9 by listing their multiples;

7 , 14, 21, 28, 35, 42, 49, 56, 63, 70

9, 18, 27, 36, 45, 54, 63, 72,

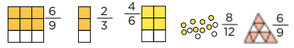
$=1\frac{10}{63}$

Equivalent Fractions

Two fractions are equivalent if they represent the same quantity

Each of these diagrams represents an equivalent amount

They all show '2 out of every 3' or $\frac{2}{3}$



If the numerator and denominator have the same multiplier, they are equivalent

$\frac{5}{7} \times5 = \frac{25}{35} \times5$ $\frac{1}{4} \times2 = \frac{2}{8} \times2$

Simplifying Fractions

$\frac{2}{4} = \frac{1}{2}$ (÷2)

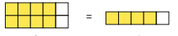
HCF of 2 and 4 is 2 so to fully simplify the fraction by dividing the numerator and denominator by 2.

$\frac{4}{12} \overset{\div2}{=} \frac{2}{6} \overset{\div2}{=} \frac{1}{3}$

$\frac{4}{12} \overset{\div12}{=} \frac{1}{3}$

Both ways get us to the right answer, just one takes a bit longer

$\frac{8}{10}$ Sometimes a picture can help visualise the problem



$\frac{8}{10} = \frac{4}{5}$

Once you cannot find a common factor, the fraction is fully simplified

$\frac{7}{10}$

This fraction is fully simplified as 7 and 10 have no common factors. We can say that 7 and 10 are COPRIME

Adding/Subtracting Fractions — Common multiples

$\frac{3}{5} + \frac{1}{10}$

10 is a multiple of 5 (5 x 2) so, using equivalent fractions we can say: $\frac{3}{5} = \frac{6}{10}$

$\frac{6}{10} + \frac{1}{10} = \frac{7}{10}$

$\frac{3}{4} - \frac{1}{12}$

12 is a multiple of 4 (4 x 3) so, using equivalent fractions we can say: $\frac{3}{4} = \frac{9}{12}$

$\frac{9}{12} - \frac{1}{12} = \frac{8}{12} = \frac{2}{3}$

Remember you must always fully simplify your fractions

$\frac{1}{2} + \frac{2}{3} + \frac{1}{6}$

Here, we know that 2 and 3 share a common multiple of 6, so we can say: $\frac{1}{2} = \frac{3}{6}$ and $\frac{2}{3} = \frac{4}{6}$

$\frac{3}{6} + \frac{4}{6} + \frac{1}{6} = \frac{8}{6}$

We need to give our answer as a mixed number

$\frac{4}{3}$ Simplify $= 1\frac{1}{3}$

Adding/Subtracting Mixed Numbers

Method 1

$1\frac{3}{4} + 2\frac{1}{2}$

We have three 'wholes' + $\frac{3}{4} + \frac{1}{2}$

$\frac{3}{4} + \frac{2}{4}$

$= \frac{5}{4}$

$= 1\frac{1}{4}$

So we have:

$3 + 1\frac{1}{4} = 4\frac{1}{4}$

Method 2

$1\frac{3}{4} + 2\frac{1}{2}$

$1\frac{3}{4} =$  $= \frac{7}{4}$

$2\frac{1}{2} =$ $= \frac{5}{2} \overset{\times2}{=} \frac{10}{4}$

$\frac{7}{4} + \frac{10}{4} = \frac{17}{4} = 4\frac{1}{4}$

How many times does 4 go into 17? 4,8,12,16,20...

4 with a remainder of 1